GEC223: FLUID MECHANICS

MODULE 4: HYDROPOWER SYSTEMS

TOPIC: IMPULSE TURBINES-PELTON WHEEL

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CONSTRUCTION AND WORKING OF A PELTON WHEEL

A pelton wheel consists of a rotor, at the periphery of which is mounted equally spaced double hemispherical/ellipsoidal buckets. Water is transferred from a high head source through a penstock which is fitted with a nozzle, through which the water flows out at a high speed jet.

A noddle spear moving inside the nozzle controls the water flow through the nozzle and also provides a smooth flow with negligible energy loss. All the potential energy (P.E.) is thus converted into kinetic energy (K.E.) before the jet strikes the buckets of the runner.

The casing prevents splashing of water and directs/discharges the water to the tail race. In order to bring the runner to rest in a short time, a brake nozzle is provided which directs the jet of water on the back of the buckets.

The jet emerging from the nozzle hits the splitter symmetrically and is equally distributed into two halves of hemispherical bucket. The angular deflection of the jet in the bucket, θ, is limited to $\sim$ 165^0-170^0.

N = Speed of wheel in rpm

D = Diameter of wheel

d = Diameter of jet

u = Peripheral (or circumferential) velocity of runner. u$=u_1 = u_1 = \frac{\pi DN}{60}$

V_1 = absolute velocity of water at inlet

V_{r1} = Jet velocity relative to vane/bucket at inlet

α = Guide angle = angle between direction of jet and direction of motion of vane/bucket.

θ = Vane angle at inlet = angle made by the relative velocity V_{r1}, with the direction of motion at inlet.

V_{w1}, V_{f1} = Components of jet velocity, V_1, in direction of motion and perpendicular to direction of motion of vane respectively.

V_{w1}= whirl velocity at inlet

V_{f1} = flow velocity at inlet

V_2= Velocity of jet leaving the vane or velocity of jet at the outlet of the vane

V_{r2}= Relative velocity of jet with respect to (w.r.t.) the outlet vane

∅ = Vane angle outlet = angle between relative velocity, V_{r2} with direction of motion of out let vane

β = Angle made by the velocity V_2, in the direction of motion of vane and perpendicular to direction of motion of vane at outlet.

V_{w2}= whirl velocity at outlet

V_{f2}= whirl velocity at outlet

Considering the inlet velocity triangle,

Since velocity triangle at inlet is a straight line,

V_{r1}= V_1- u_2= V_1- u (Since u_1=u_2=u)

V_{w1}= V_1 and α = 0 and θ = 0

Considering the outlet velocity triangle,

V_{r2} = KV_{r1}

Where K = blade friction coefficient, usually < 1. This is because the buckets are usually not smooth.

When bucket is perfectly smooth, K= 1

V_{w2}= V_{r2}cos∅-u_2 = V_{r2}cos∅ - u since $u_1 = u_2$ =u (when β< 90^0)

Depending on the magnitude of the peripheral speed (u), the unit may have a slow, medium or fast runner and the blade angle β and V_{w2} will vary as follows:

i. Slow runner $\beta < 90^0$; V_{w2} is +ve
ii. Medium runner $\beta = 90^0$; $V_{w2} = 0$
iii. Fast runner $\beta > 90^0$; V_{w2} is +ve

The force exerted by the jet of water in the direction of motion, F = $\rho a V_1\,(V_{w1} + V_{w1})$

Where ρ = mass density of water

A = area of jet of water = $\frac{\pi d^2}{4}$

Work done by the jet on runner per second = F x u = $\rho a V_1\,(V_{w1} + V_{w1})$ x u

Work done per second per unit weight of water striking the buckets = $\frac{\rho a V_1\,(V_{w1}+V_{w1})\text{ x u}}{\rho a V_1\ x\ g}$

$$= \frac{1}{g}[V_{w1} + V_{w1}]\text{u}$$

The energy supplied to the jet at inlet in form of K.E. = ½ m$V_1{}^2$

Recall, m = ρQ and Q = aV_1, ∴ m = ρaV_1

∴ K.E. of jet per second = ½ (ρaV_1) x $V_1{}^2$

∴ Hydraulic Efficiency, $\eta_h = \frac{Work\ done\ per\ second}{K.E. of\ jet\ pers\ second} = \frac{\rho a V_1\,(V_{w1}+V_{w1})\text{ x u}}{½\,(\rho a V_1)\text{ x }V_1{}^2}$

∴ Hydraulic Efficiency, $\eta_h = \frac{2(V_{w1}+V_{w1})\text{ x u}}{V_1{}^2}$ ________Equation 1

From outlet and inlet velocity triangles,

$V_{w1} = V_1$; $V_{r1} = V_1 - u_1 = V_1$ - u

$V_{w2} = V_{r2}\cos\phi - u_2 = V_{r2}\cos\phi$ – u

Since V_{r2} = kV_{r1} and $V_{r1} = V_1$ - u

$V_{w2} = kV_{r1}\cos\phi$ – u = k (V_1-u)$\cos\phi$-u

Substituting for V_{w1} and V_{w2} in equation 1 gives,

$\eta_h = \frac{2[V_1 + k(V_1 - u)\cos\phi - \text{u}]u}{V_1{}^2}$

Rearranging gives,

$\eta_h = \frac{2[V_1 - u + k(V_1 - u)\cos\phi]u}{{V_1}^2} =$

$\boldsymbol{\eta_h} = \frac{\mathbf{2[(V_1 - u)(1 + k\cos\phi)]u}}{\mathbf{{V_1}^2}}$ ______________Equation 2

The hydraulic efficiency will be maximum for given value of V_1 when $\frac{d}{du}(\eta_h)$ = 0

This implies $\frac{d}{du}\left[\frac{\mathbf{2[(V_1 - u)(1 + k\cos\phi)]u}}{\mathbf{{V_1}^2}}\right]$ = 0

Or $\frac{\mathbf{2(1 + k\cos\phi)u}}{\mathbf{{V_1}^2}}$ **x** $\frac{d}{du}(V_1 u - u^2)$ = 0

Since $\frac{\mathbf{2(1 + k\cos\phi)u}}{\mathbf{{V_1}^2}} \neq \mathbf{0}$

$\therefore \frac{d}{du}(V_1 u - u^2)$ = 0

This implies V_1-2u = 0

$\therefore$ **V_1= 2u and u =** $\frac{\mathbf{V_1}}{\mathbf{2}}$________________Equation 3

This implies that hydraulic efficiency of a pelton wheel is maximum when the velocity of the wheel is half the velocity of jet of water at inlet.

The maximum hydraulic efficiency is obtained by substituting u = $\frac{V_1}{2}$ in equation 2.

$(\boldsymbol{\eta_h})_{\boldsymbol{max}} = \frac{\mathbf{2\left[(V_1 - \frac{V_1}{2})(1 + k\cos\phi)\right]\frac{V_1}{2}}}{\mathbf{{V_1}^2}} = \frac{\mathbf{2\left[\frac{V_1}{2}(1 + k\cos\phi)\right]\frac{V_1}{2}}}{\mathbf{{V_1}^2}}$

$\therefore (\boldsymbol{\eta_h})_{\boldsymbol{max}} = \frac{\mathbf{1 + k\cos\phi}}{\mathbf{2}}$ __________________Equation 4

Assuming no friction, k = 1 and

$\therefore (\boldsymbol{\eta_h})_{\boldsymbol{max}} = \frac{\mathbf{1 + \cos\phi}}{\mathbf{2}}$ ____________________Equation 5

DEFININTIONS OF HEADS AND EFFICIENCIES

1. Gross Head: The gross (total) head is the difference between the water level at the reservoir (known as the tail race) and the water level at the tail race. It is denoted as Hg.
2. Net or effective Head: The head available at the inlet of the turbine. It is denoted as H, where H = Hg - h_f – h
   Where h_f = total loss of head between the head race and entrance to the turbine defined as

   $$h_f = \frac{4fLV^2}{D\ X\ 2g}$$

   where L = length of penstock
   D = diameter of penstock
   V = velocity of penstock
   h = height of nozzle above the tail race

3. Efficiencies: In turbines, there are four kinds of efficiency:

i. Hydraulic Efficiency, η_h : This is the ratio of power developed by the runner to the power supplied by the jet at the entrance to the turbine.

$$\eta_h = \frac{Power\ developed\ by\ the\ runner}{Power\ supplied\ by\ the\ jet\ at\ turbine\ inlet}$$

$$= \frac{\rho Q_a\,(V_{w1} \pm V_{w2})u}{wQ_aH} = \frac{\frac{w}{g}Q_a\,(V_{w1} \pm V_{w2})u}{wQ_aH}$$

$$\eta_h = \frac{(V_{w1} \pm V_{w2})u}{gH}$$

Where V_{w1} = whirl velocities atinlet

V_{w1} = whirl velocities at outlet

H = Net head on the turbine

Q_a = Actual flow rate to turbine runner (bucket)

Runner head or Euler head, $H_r = \frac{(V_{w1} \pm V_{w2})u}{g}$

Where H_r represents the energy transferred per unit weight of water.

H - H_r = ΔH = Hydraulic losses within the turbine.

$$\therefore \boldsymbol{\eta_h} = \frac{\boldsymbol{H_r}}{\boldsymbol{H}}$$

ii. Mechanical Efficiency, η_m = Ratio of power obtained from the shaft of the turbine to the power developed by the runner.

$$\eta_m = \frac{Power\ available\ at\ the\ turbine\ shaft}{Power\ developed\ by\ the\ turbine\ runner}$$

$$\eta_m = \frac{Shaft\ Power}{Bucket\ Power}$$

$$\eta_m = \frac{P}{wQ_a\left(\frac{V_{w1}+V_{w2}}{g}\right)u}$$

Since $H_r = \frac{(V_{w1} \pm V_{w2})u}{g}$,

$$\boldsymbol{\eta_m} = \frac{\boldsymbol{P}}{\boldsymbol{wQ_aH_r}}$$

η_m lies between 97-99%.

iii. Volumetric Efficiency: Ratio of volume of water actually striking the runner to the volume of water supplied by the jet to the turbine.

$$\eta_v = \frac{Volume\ of\ water\ striking\ the\ runner\ (Q_a)}{Total\ water\ supplied\ by\ the\ jet\ to\ turbine\ (Q)}$$

For pelton turbines, $\eta_v \cong 0.97\text{-}0.99$.

iv. Overaal Efficiency: Ratio of power available at the turbine shaft to the power supplied by the water jet.

$$\eta_o = \frac{Power\ available\ at\ the\ turbine\ shaft}{Power\ available\ from\ the\ water\ jet}$$

$$\eta_o = \frac{Shaft\ Power}{Water\ Power} = \frac{P}{wQH}$$

Where Q = total discharge in m^3/s supplied by the jet of water.

For a pelton wheel, overall efficiency is between 0.85-0.90

$$\eta_o = \eta_h \times \eta_m \times \eta_v$$

$$\eta_o = \frac{H_r}{H}\ x\ \frac{P}{wQ_aH_r}\ x\ \frac{Q_a}{Q} = \frac{P}{wQH}$$

$$\boldsymbol{\eta_o} = \frac{\boldsymbol{P}}{\boldsymbol{wQH}}$$

∴ Power output from turbine alone P = Wqh x η_o

Considering the efficiency of the generator as η_g,

Power output of the hydrounit (turbine + hydrogenerators), P = (wQH) x η_o x η_g

The product η_o x η_g = Hydroelectric plant efficiency.

DESIGN ASPECTS OF PELTON WHEEL

1. Velocity of jet at inlet, $\boldsymbol{V_1 = C_v\sqrt{2gH}}$

Where C_v = coefficient of velocity between 0.98-0.99.

H = Net head on turbine

2. Velocity of wheel, **U =** $\boldsymbol{K_u\sqrt{2gH}}$

Where K_u = Speed ratio between 0.43-0.48.

3. Angle of deflection of ject through the buckets: Ranges between 165^0-175^0.

4. Mean diameter or Pitch diameter of the pelton wheel D is given by U = $\frac{\pi DN}{60}$

$$\therefore \mathbf{D} = \boldsymbol{\frac{60U}{\pi N}}$$

5. Jet ratio (m) = Ratio of pitch diameter of pelton wheel to diameter of the jet (d).

m = $\boldsymbol{\frac{D}{d}}$ (lies between 11-16 for maximum hydraulic efficiency). In practice, m = 12 is adopted.

6. Number of Jets: Practically, ≤ 2 jets per runner for a vertical runner and ≥ 4 jets per runner for a horizontal jet.

Number of Jets = Total flow rate through the turbine divided by the rate of water through a single jet.

7. **Number of Bucktes (Z) = 15 +** $\boldsymbol{\frac{D}{2d}}$ **= 15 + 0.5**$\boldsymbol{\frac{D}{d}}$

Example 1

A pelton wheel running at 480 r.p.m. and operating under an available head of 420m is required to develop 4800KW. There are two equal jets and the bucket deflection angle is 165^0. The overall efficiency is 85% when the water is discharged from the wheel in a direction parallel to the axis of rotation. The coefficient of velocity of nozzle is 0.97 and the blade speed ratio is 0.46. The relative velocity of water at exit from the bucket is 0.86 times the relative velocity at inlet. Calculate the following:

i. Cross-sectional area of each jet
ii. Bucket pitch circle diameter, and
iii. Hydraulic efficiency of the turbine

Solution

Speed of the wheel, N = 480 r.p.m.

Available head, H = 420m

Shaft power, P = 4800KW

Angle of deflection of jet = 165^0

Overall efficiency, η_o = 85%

Coefficient of velocity of nozzle, C_v = 0.97

Blade speed ratio, K_u = 0.46

Relative velocity of water at exit = 0.86 times the velocity at inlet

i. Cross sectional area of each jet

Shaft power, P = wQH x η_o

4800 = 9.81 x Q x 420 x 0.85

∴ Total discharge through the wheel, Q = $\frac{4800}{9.81 \ x \ 420 \ x \ 0.85}$ = $1.37 m^3/s$

Velocity of jet , $V_1 = C_v\sqrt{2gH} = 0.97\sqrt{2 \ x \ 9.81 \ x \ 420}$ = 88.05m/s

Total discharge, Q = No. of jets x area of each nozzle (a) x velocity of jet, V_1

1.37 = 2 x a 88.05

a = $\frac{1.37}{2 \ x \ 88.05}$ = **$7.779 \times 10^{-3} m^2$**

ii. Bucket pitch circle diameter, D

Velocity of bucket, u = $K_u\sqrt{2gH}$ = $0.46\sqrt{2\ x\ 8.81\ x\ 420}$ = 41.76m/s

U = $\frac{\pi DN}{60}$. This implies, 41.76 = $\frac{\pi D\ x\ 480}{60}$

∴ D = $\frac{41.76\ X\ 60}{\pi\ X\ 480}$ = **1.66m**

iii. Hydraulic efficiency of turbine, η_h

η_h = $\frac{2(V_1-u)(1+kcos\emptyset)u}{{V_1}^2}$ where Ø = 180^0-165^0 = Blade angle at exit

η_h = $\frac{2(88.05-41.76)(1+0.86xcos15^0)x\ 41.76}{88.05^2}$

η_h = 0.913 = **91.3%**

Example 2

The water available for a pelton wheel is 4m^3/s and the total head from the reservoir to the nozzle is 250m. The pipe is 3km long. The efficiency of transmission through the pipeline and the nozzle is 91% and the efficiency of each runner is 90%. The velocity coefficient of each nozzle is 0.975 and coefficient of friction "4f" for the pipe is 0.0045. Determine:

i. The power developed by the turbine
ii. Diameter of the jet
iii. Diameter of the pipeline

Solution

Rate of flow, Q = 4m^3/s

Total or gross head, H_g = 250m

Total number of jets = 2 x 2 = 4

Length of pipe, L = 3km = 3000m

Efficiency of transmission, η = 91% = 0.91

Efficiency of each runner, η_h = 90% = 0.90

Coefficient of friction for the pipe, 4f = 0.0045

Coefficient of velocity of each nozzle = 0.975

1) Power developed by the runner

Efficiency of power transmission, $\eta = \frac{H_g - h_f}{H_g}$

Where h_f = loss of head due to friction

$0.91 = \frac{250 - h_f}{250}$

h_f = 250- (250 x 0.91) = 22.5m

Net head on the turbine, H = $H_g - h_f$ = 250-22.5 = 227.5m

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.975\sqrt{2 \; x \; 9.81 \; x \; 227.5}$ = 65.14m/s

Water power = Kinetic energy of jet

i.e. ½ $mV_1{}^2$ = ½ $\rho Q V_1{}^2$ = ½ x 1000 x 4 x 65.14^2 = 8486439Nm/s = 8486439W

Water power = 8486.44KW

Hydraulic Efficiency, $\eta_h = \frac{Power \; developed \; by \; turbine}{Water \; power}$

$0.91 = \frac{Power \; developed \; by \; turbine}{8486.44}$

∴ Power developed by turbine = 0.9 x 8486.44 = **7637.8KW**

2) Diameter of jet, d

Discharge per jet, q = $\frac{Total \; discharge}{No.of \; jets} = \frac{4}{4}$ = 1.0m^3/s

$q = \left(\frac{\pi d^2 x \, V_1}{\pi \; x \; 65.14}\right)^{1/2} = \frac{\pi d^2 x \; 65.14}{4}$

∴ $d = \left(\frac{1 \; x \; 4}{\pi \; x \; 65.14}\right)^{1/2}$ = **0.14m**

3) Diameter of pipeline, D

Head lost due to friction, $h_f = \frac{4fLV^2}{D \; x \; 2g}$

Where V = velocity through pipe = $\frac{Q}{Area} = \frac{Q}{\frac{\pi D^2}{4}} = \frac{4Q}{\pi D^2}$

Substituting for V in h_f gives

$$h_f = \frac{0.0045\ x\ 3000\ x\left(\frac{4Q}{\pi D^2}\right)}{D\ \ x\ 2g}$$

$$22.5 = \frac{0.0045\ x\ 3000\ x\ 16Q^2}{D\ x\ 2\ x\ 9.81\ x\ \pi^2\ x\ D^4} = \frac{0.0045\ x\ 3000\ x\ 16\ x\ 4^2}{D^5\ x\ 2\ x\ 9.81\ x\ \pi^2} = \frac{17.85}{D^5}$$

$$D^5 = \frac{17.85}{22.5}$$

$$\therefore \boldsymbol{D} = \left(\frac{\mathbf{17.85}}{\mathbf{22.5}}\right)^{\mathbf{1/5}} = \underline{\mathbf{0.955m}}$$

Example 3

A pelton wheel nozzle, for which C_v is 0.97, is below the water surface of a lake. The jet diameter is 80mm, the pipe diameter is 0.6m, its length is 4km, and f is 0.032 in the formula, $\eta_f = \frac{fLV^2}{D\ x\ 2g}$. The buckets deflect the jet through 165^0 and they run at o.48times the jet speed, bucket friction reducing the velocity at outlet by 15% of the relative velocity at inlet. Mechanical efficiency is 90%. Determine:

i. The flow rate
ii. Shaft power developed by the turbine

Solution

Coefficient of Velocity, C_v = 0.97

Gross head, H_g = 400m

Diameter of jet, d = 80mm= 0.08m

Diameter of pipe, D= 0.6m

Length of pipe, L = 4km = 4000m

Friction factor, f = 0.32

Angle, $\varphi = 180^0 - 165^0 = 15^0$

Bucket speed, u = 0.48 times the jet speed

Relative velocity at the outlet (V_{r2}) = 0.85 times relative velocity at inlet V_{r1}

1) Flow rate, Q

V = Velocity of water in pipe

V_1 = Velocity of jet of water

Using equation of continuity,

AV = a V_1

Where A = Area of pipe and a = area of jet

$\frac{\pi D^2}{4}$ x V = $\frac{\pi d^2}{4}$ x V_1

∴ V = $\frac{d^2}{D^2}$ x V_1 = $\frac{0.08^2}{0.6^2}$ x V_1 = $0.0177V_1$

Applying Bernoulli's equation to free water surface in the reservoir and the outlet of the nozzle,

Head at reservoir = Kinetic head of jet of water + head lost to friction in pipe + head lost in nozzle

$H_{reservoir}$ = $\frac{V_1{}^2}{2g}$ + $\frac{fLV^2}{D\ x\ 2g}$ + h_{nozzle}___________equation 1

Let $(V_1)_{th}$ = Theoretical velocity at outlet of nozzle

V_1= Actual velocity of jet of water

$\frac{V_1}{(V_1)_{th}}$ C_v or $(V_1)_{th}$ = $\frac{V_1}{C_v}$

Head lost in nozzle = Head corresponding to $(V_1)_{th}$- head corresponding to V_1.

$$= \frac{(V_1)_{th}{}^2}{2g} - \frac{V_1{}^2}{2g} = \left(\frac{V_1}{C_v}\right)^2 \text{ x } \frac{1}{2g} - \frac{V_1{}^2}{2g}$$

$$= \frac{V_1{}^2}{2g}\left(\frac{1}{C_v{}^2} - 1\right)$$

Substituting this in equation 1 above gives,

$$H_{reservoir} = \frac{V_1{}^2}{2g} + \frac{fLV^2}{D\ x\ 2g} + \frac{V_1{}^2}{2g}\left(\frac{1}{C_v{}^2} - 1\right)$$

$$400 = \frac{V_1{}^2}{2g} + \frac{0.032\ x\ 4000V^2}{0.6\ X\ 2\ X\ 9.81} + \frac{V_1{}^2}{2\ x\ 9.81\ x\ 0.97^2}$$

$$400 = \frac{0.032\ x\ 4000\ (0.0177V_1)^2}{0.6\ X\ 2\ X\ 9.81} + \frac{V_1{}^2}{2\ \ x\ 9.81\ x\ 0.97^2}$$

$$400 = 0.0034V_1{}^2 + 0.054V_1{}^2 = 0.0574V_1{}^2$$

$$V_1 = \left(\frac{400}{0.0574}\right)^{1/2} = 83.48\text{m/s}$$

∴ Flow rate, Q = Area of jet (a) x Velocity of jet (V) = Av = $\frac{\pi}{4}$ x 0.08^2 x 83.48

Q = <u>0.419m³/s</u>

2) Shaft power

Velocity of bucket, u_1 = 0.48V_1 = 0.48 X 83.48 = 40.07M/S

From the inlet velocity triangle,

V_{r1} = V_1 - u_1 = 83.48-40.07 = 43.4m/s

V_{w1} = V_1 = 83.48m/s

From the outlet velocity triangle,

V_{r2} = 0.85V_{r1} = 0.85 x 43.4 = 36.89m/s

V_{w2} = u_2 -V_{r2}cosØ = 40.07-36.89 x cos15^0 = 4.44m/s

Mechanical efficiency, $\eta_m = \frac{Shaft\ power}{Power\ given\ to\ runner}$

∴ Shaft power= η_m x Power given to runner

Power given to runner = $\frac{wQ}{g}(V_{w1} - V_{w2})x\ u$

Note: -ve sign is given here because β > 90^0.

∴ Shaft power = η_m x $\frac{wQ}{g}(V_{w1} - V_{w2})x\ u_1$ = 0.9 x $\frac{9.81}{9.81}$x0.419(83.48-4.44) 40.07

∴ **Shaft power = 1194.3KW**

Example 4

The following data relate to a pelton wheel:

Head_______________72m

Speed of wheel_____________240rpm

Shaft power of wheel_______115KW

Speed ratio________________0.45

C_v _________________________0.98

Overall efficiency____________0.85

Design the Pelton wheel

Solution

Effective head, H = 72m

Speed of wheel, N = 240rpm

Shaft power, P = 115KW

Speed ratio, K_u = 0.45

C_v = 0.98

Overall efficiency, η_0= 85%

1) Diameter of wheel, D

Velocity of jet, V_1 = $C_v\sqrt{2gH}$ = $0.98\sqrt{2 \, x \, 9.81 \, x \, 72}$ = 36.8m/s

Bucket velocity, u (=u_1 = u_2) = K_u x V_1 = 0.45 x 36.8 = 16.56m/s

u = $\frac{\pi DN}{60}$. This implies, D = $\frac{60u}{\pi N}$ = $\frac{60 \, x \, 16.56}{\pi \, x \, 240}$ = 1.32m

∴ **<u>Diameter of wheel, D = 1.32m</u>**

2) Diameter of jet, d

Overall efficiency, η_0= $\frac{Shaft \, power}{Water \, power}$ = $\frac{P}{wQH}$

η_0= $\frac{115}{9.81 \, x \, Q \, x \, 72}$

Q = $\frac{115}{0.85 \, x \, 9.81 \, x \, 72}$ = 0.1915m^3/s

Q = Area of jet x jet velocity

0.1915 = $\frac{\pi d^2}{4}$ x V_1 = $\frac{\pi d^2}{4} \, x \, 36.8$

d = $\left[\frac{0.1915 \, x \, 4}{\pi \, x \, 36.8}\right]^{1/2}$ = 0.0814m = 81.4mm

∴ **<u>diameter of jet, d = 81.4mm = 0.0814m</u>**

3) Size of buckets

Width of bucket, B = 3d to 4d ∴ choosing B = 3.5d

B = 3.5 x 81.4 = 285mm

Radial length of bucket, L = 2d to 3d ∴ choosing L = 2.5d,

L = 2.5 x 81.4mm = 203.5mm

Depth of bucket, T = 0.8d to 1.2d ∴ choosing T = 1.0d,

T = 1 x 81.4 = 81.4mm

4) Number of buckets on the wheel, Z

Z = 15 + $\frac{D}{2d}$ = 15 + $\frac{1.32 \, x \, 1000}{2 \, x \, 81.4}$ = 23

<u>Number of buckets, Z = 23</u>

Example 5

A pelton wheel of 1.1 mean diameter works under a head of 500m. The deflectionof jet is 165^0 and its relative velocity is reduced over the bucket by 15% due to friction. If the diameter of jet is 100mm and the water is to leave the bucket without any whirl, determine:

i. Rotational speed of wheel
ii. Ratio of bucket speed to jet velocity
iii. Impulsive force and power developed by the wheel
iv. Available power (that is water power)
v. Power input to buckets
vi. Efficiency of the wheel with power input to bucket as reference point.

Solution

Main bucket diameter, D = 1.1m

Net head, H = 500mm

Jet deflection = 165^0

Reduction of relative velocity due to friction = 15%

Jet diameter, d = 100mm = 0.1m

C_v = 0.97

i. Rotational Speed of wheel

Velocity of jet, $V_1 = C_v\sqrt{2gH}$ = $0.97\sqrt{2\ x\ 9.81\ x\ 500}$ = 96.07m/s

Bucket speed, u_1 = u_2 = u

Relative velocity at inlet, V_{r1} = $V_1 - u_1$ = 96.07-u

Relative velocity at outlet, V_{r2}= 0.85V_{r1} = 0.85(96.07-u) _____________equation 1

Blade angle at exit, Ø = 185^0-165^0 =15^0

Since jet leaves bucket without whirl, $\beta = 90^0$

V_{r2}cosØ = u from outlet velocity triangle

This implies u = V_{r2}cos15^0 _______equation 2

Substituting for equation 1 in equation 2 gives,

u = 0.85 (96.07-u) cos15^0 = 0.85(96.07-u)0.966

78.88-0.812u =u

U +0.821u = 78.88

1.821u = 73.88

u = $\frac{73.88}{1.821}$ = 43.31m/s

Recall, u = $\frac{\pi DN}{60}$. This implies, 43.31 = $\frac{\pi\ x\ 1.1\ x\ N}{60}$

∴ N = Rotational speed of wheel = $\frac{43.31\ x\ 60}{\pi\ x\ 1.1}$ = **752rpm**

ii. Ratio of bucket speed to jet velocity, $\frac{u}{V_1}$ = $\frac{43.31}{96.07}$ = **0.45**

iii. Impulsive force and power developed by the wheel

Discharge through the wheel, Q = $\frac{\pi}{4}$ x d^2 x V_1 = $\frac{\pi\ x\ 0.1^2}{4}$ x 96.07 = 0.7545m^3/s

Impulsive force on buckets, F = $\rho Q(V_{w1} \pm V_{w2})$ Since V_{w2}= 0

F = $\rho Q(V_{w1})$ = 1000 x 0.7545 x 96.07 = **72484.8N**

Power developed by wheel = F X u = 72484.8 x 43.31 = 3139316.7Nm/s = **3139.3KW**

iv. Available power = w QH = 9.81 x 0.7545 x 500 = **3700.8KW**

v. Power input to buckets = ½ m$V_1{}^2$= ½ ρQx $V_1{}^2$= ½ x 1000 x 0.7545 x 96.07^2 = 3481808W =**3481.8KW**

vi. Efficiency of wheel, η_{wheel} = $\frac{Power\ developed\ by\ wheel}{Power\ input\ to\ buckets}$ = $\frac{3139.3}{3481.3}$ = 0.9016 = **90.16%**

Note: Head loss in nozzle, H_{nozzle} = (1-${C_v}^2$) H =

Head loss in buckets, $H_{buckets}$ = $\frac{{V_{r1}}^2}{2g}$ (1-K^2)

Power loss in nozzle and buckets = wQH_L where H_L= H_{nozzle} + $H_{buckets}$